Global Link Singapore 2018 : Abstract for Science Project

Submission Deadline：04 May, 2018

Study Area　*Please mark a circle on your subject

Physics　Chemistry　Medical Science･Biology　Earth Science･Geoscience Mathematics･Informatics･Computer Science　Others(　　)

Category　*Please mark a circle on your presentation style

Oral Presentation　/　Poster Presentation

Presenter(s)

【School Name】 National Junior College
【Project Member(s)】 *Name removed to protect the identity of student

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Abstract of Presentation

【 Title 】 Scrutiny of Winners and Tournaments

【 Introduction/Background/Motivation 】

In almost every sport and game, the best team or player is determined through tournaments. Offhandedly, the ideal situation seems to be letting every team play with each other the same number of times and determine the best team by selecting the one with the most number of wins. This would then encounter the issue about time as the round-robin format takes a long time, which would make holding tournaments under time constraint or tournaments with a large number of teams difficult. For example, in the NBA season, all thirty teams play with each other more than once (amounting to eighty two games for each team). This is very fair as all teams play the same number of games and play directly against each other. However, it is time consuming. On the other hand, another method to hold tournaments is to select teams to play against each other where only the winner proceeds to the next round. This is efficient as it consumes very little time. However, there are a few issues in this game plan. First, teams which have climbed up further in the rounds will play more games than those which have not, which affects the playoffs standing due to the difference in strength. Second, the champion team may win in the finals despite not establishing a 'winning' relationship against the most number of teams in the seasons.

【 Research Purpose/Problem Statement 】

In this project, we aim to find an optimum way to plan tournaments so as to (1) ensure a fair system where every team play the same number of games, (2) reduce the time involved and (3) decide an effective definition of best team (or to break tie).

Small-scale tournaments often make use of a tournament format wherein random teams are chosen to play against each other and only the winners are able to proceed to the next round. This prioritises efficiency in tournaments, and compromises fairness. Cases of unfairness may surface in cases when the 2 strongest teams are picked to play against each other in the first match, thereby eliminating the 2nd strongest team and preventing it from proceeding. To solve this problem, we will find a fair yet efficient tournament format, specifically targeting small-scale tournaments held under time constraint.

【 Study Plan/Approach 】

To investigate the optimum way, we first explore the concept of tournament graphs, emperors and kings. To reduce the amount of time tournaments span over, we switched out the concept of complete graphs for regular graphs. Then, we devised a new family of graphs that satisfies the requirements and found an algorithm to generate the family of graphs. Next we went on to investigate the relationships presented in the graphs.

【 Results and Discussion 】

We have created a tournament format where represented by a family of graphs in which graphs (a) are regular and (b) satisfy that for any two vertices u, v in K, $d(u, v) \leq 2$. This means that a vertex can reach every other vertex in at most two steps. This reduces the total number of edges in the graph, thus reducing the time required for the tournament.

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The graphs can be formulated using two algorithms, which were then proven:

1. The graphs in *K* with *n* vertices can be formed using "skip $\frac{n-2}{8}$ cycle", "skip $\frac{n-2}{8}-1$ cycle", …, and "skip $\frac{n-2}{4}$ cycle".
2. The number of "skip *x* cycles" is $\frac{n-10}{8}$.

We go on to investigate the possible outcomes of the tournament planned out using a graph in K. There are several cases wherein the winner can not be determined where there exists:

A directed cycle. Given any directed cycle, it will not be possible to determine the winner.

Contradictory results between direct and indirect relationships.

Unestablished results between vertices. When 2 disjoint vertices win or lose to a mutual vertex, which connects the two vertices, the relationship between the two vertices would fail to be established.

Multiple vertices with maximum weight. A tie also arises in a scenario in which a winner cannot be determined is one where there is more than 1 vertex with the maximum directed weight. Directed weight refers to the number of edges it is connected to that are directed away from it.

To avoid these problems, we create a table to express all the relationships between team. The relationships can be represented by 1, 0, -1. 1 denotes a win, 0 denotes a tie or an inconclusive relationship due to faults, while -1 denotes a lose. In the case of a tie, we will resolve the tie by holding a tiebreaker match, reusing the same tournament format.

【Future Study Plan】

In future, we aim to find the total number of different paths that connect any 2 vertices, to more fairly tabulate the scores, easily identify all direct and indirect relationships and eliminate all the inconclusive results. We also aim to find more configurations of K to find even more efficient configurations to expand the types of K graphs that can be used, so as to find the most ideal. A programme can also be coded to automatically compile all the results and determine the winners.

【References】

Yu, Y., Di, J., & Li, M. (September 2006). Kings in Tournaments. Mathematical Medley, 33(1), 28-32. Retrieved November 17, 2017, from https://sms.math.nus.edu.sg/smsmedley/Vol-33-1/Kings%20in%20Tournaments%20(Yu%20Yibo,%20Di%20Junwei,%20Lin%20Min).pdf.

Yu, Y. (December 2006). Kings in Tournaments (2) . Mathematical Medley , 33(2), 15-18. Retrieved November 17, 2017, from https://sms.math.nus.edu.sg/smsmedley/Vol-33-2/Kings%20in%20Tournaments%202%20(Yu%20Yibo).pdf.